The Stochastic Wave Equation with an Interval Valued Parameter

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Abstract

We consider the solution u_c of the stochastic wave equation in three space dimensions

$$\partial_t^2 u_c - c^2 \Delta u_c = \dot{W} \qquad\qquad u_c : \Omega \to \mathcal{S}'\left(\mathbb{R}^4\right)$$

denoting by $\dot{W}$ the white noise with support in $[0,\infty)\times\mathbb{R}^3$. It is a generalized stochastic process on a probability space (Ω, Σ, μ). A suitable choice for Ω is the space of tempered distributions $\mathcal{S}'(D)$.

Modelling the parameter c as an interval means to investigate the function

$$\begin{aligned} X : \Omega &\to P\left(\mathcal{S}'\left(\mathbb{R}^4\right)\right) \\ \omega &\mapsto \{u_c(\omega), c_1 \le c \le c_2\} \end{aligned}$$

In this contribution we show that X fulfils the Borel measurability condition

$$X^-(B) := \{\omega \in \Omega : X(\omega) \cap B \neq \emptyset\} \in \mathcal{B}(\Omega) \qquad\qquad \forall B \in \mathcal{B}\left(\mathcal{S}'\left(\mathbb{R}^4\right)\right)$$

and therefore is a random set in the general sense of Molchanov [1].

Keywords. Stochastic PDEs, Random Sets, Distributions.

References

[1] Ilya Molchanov. *Theory of random sets.* Probability and its Applications (New York). Springer-Verlag London Ltd., London, 2005.