First steps towards Little's Law with imprecise probabilities

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Abstract

In this research we take the first steps towards approaching the (distributional version of) Little's Law [1, 5, 6] from an imprecise-probabilistic point of view. We examine the law for a discrete-time, single-server queue where the arrivals and the servicing (departures) happen according to imprecise Bernoulli processes: forward irrelevant arrivals [3, 4] occur at each discrete time point with probability interval $[\underline{a}, \overline{a}]$ and, similarly, forward irrelevant departures occur at each discrete time point with probability interval $[\underline{d}, \overline{d}]$. Arrivals and departures are assumed to be epistemically independent [8].

We make two additional assumptions regarding the properties of the queue as well. The first one is that upon arriving, an item needs to remain in the queue till served. And secondly, departure is characterised by the FIFO (first in first out) property. These assumptions allow us to get closer to the distributional version of the Little's Law, as we can use them to relate (at any time point) the distribution of the size of the queue with the time spent in the queue.

We present our results using the framework of coherent lower and upper previsions [8]. Our main result is a relation between the lower (and upper) prevision of the waiting time D_t of the last item in the queue and the lower (and upper) prevision of the number L_t of items in the queue at any given time point t. More specifically, at any time t, we get $\underline{P}(L_t) = \overline{d}\underline{P}(D_t)$ and $\overline{P}(L_t) = \underline{d}\overline{P}(D_t)$. As a consequence, we find that this result also holds when, rather than forward irrelevance, we impose more stringent independence assumptions on the departure process, such as epistemic independence [7, 8], or strong independence [2].

We also address some questions related to the limit behaviour of the queuing system. What does the (imprecise) stationary distribution of the number of items look like? How can we use our main result above to derive the stationary distribution of the waiting time? And finally, how is this stationary behaviour influenced by the arrival process?

Keywords. Little's Law, Bernoulli processes, coherent lower (and upper) previsions, forward irrelevance, epistemic independence.

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