Parameter Dependent Uncertainty in Limit State Functions

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Abstract

In *reliability theory* an engineering system is given together with its *limit state function* $g : \mathcal{X} \subseteq \mathbb{R}^n \to \mathcal{Y} \subseteq \mathbb{R} : x \to y = g(x)$ where $x = (x_1, \ldots, x_n) \in \mathcal{X}$ is a vector of basic variables (such as material properties, loads, etc.) and where $g(x) \leq 0$ means failure of the system. Then the *probability* p_f *of failure* of the system is obtained by

$$p_f = P(g(X) \leq 0) = \int_{\mathcal{X}} \chi(g(x) \leq 0) f^X(x) \,\mathrm{d}x \tag{1}$$

where f^X is the joint density function of the random variables $X = (X_1, \ldots, X_n)$ and where χ is the indicator function. In the case of scarce information about the values of the basic variables x and the behavior of the system it is neither sufficient to model the uncertainty of x by a single probability density f^X nor to describe the system's reliability by a single deterministic limit state function g. A better way to model the uncertainty of the basic variables and the uncertainty in the limit state function is to use *sets of probability measures* (*credal sets*) which will result in *upper probabilities* $\overline{p}_f$ of failure. In our approach we parameterize the limit state function by additional parameters $z = (z_1, \ldots, z_m) \in \mathcal{Z} \subseteq \mathbb{R}^m$ using a function $h : \mathcal{X} \times \mathcal{Z} \to \mathcal{Y} : (x, z) \to h(x, z)$ where $h(x, z) \leq 0$ again means failure. Then a function $g_z : \mathcal{X} \to \mathcal{Y} : x \to g_z(x) = h(x, z)$ is one of the available limit state functions specified by a parameter value z. Both the basic variables x and these new additional parameters z are uncertain which means that we are not only uncertain in the choice of the values of the basic variables but also in the choice of an appropriate limit state function g_z. In [1] we assumed that the corresponding random variables X and Z are always independent and discussed the meaning of different notions of independence for sets of probability measures in the context of limit state functions. Such an assumption may be too restrictive, especially in cases where the preference we have for some limit state functions g_z may change with the values of the basic variables x.

As an extension of [1] the poster presentation is devoted to *parameter dependent uncertainty in limit state functions*. Our starting point is the formula $p_f = \int_{\mathcal{X}} \int_{\mathcal{Z}} \chi(h(x, z) \leq 0) f^{Z|x}(z) \,\mathrm{d}z\, f^X(x) \,\mathrm{d}x$ for the probability of failure with conditional density $f^{Z|x}$ of Z given x. We extend the probability of failure p_f to a mapping

$$p_f(a, b) = \int_{\mathcal{X}} \int_{\mathcal{Z}} \chi(h(x, z) \leq 0) f^{Z|x}_{b(x)}(z) \,\mathrm{d}z\, f^X_a(x) \,\mathrm{d}x \tag{2}$$

depending on parameters where $a = (a_1, \ldots, a_{n_a}) \in \mathcal{A} \subseteq \mathbb{R}^{n_a}$ are the parameters of the density function f^X_a describing the uncertainty of the basic variables. The parameters b depend here on the basic variables x which means that b is a function $b : \mathcal{X} \to \mathcal{B} \subseteq \mathbb{R}^{n_b} : x \to b(x) = (b_1(x), \ldots, b_{n_b}(x))$ which provides parameter values $b_1(x), \ldots, b_{n_b}(x)$ to the densities of $Z|x$ for given x while in [1] b did not depend on x because of the independence of X and Z. In a next step a and b are assumed to be uncertain and sets or random sets are used to describe their uncertainty which leads to *sets of probability measures* for the random variables X and $Z|x$ and to *upper probabilities* $\overline{p}_f$ of failure. Further we will present an alternative approach using uncertain *random fields* defined on the set of basic variables to describe the uncertainty of the limit state function.

Keywords. Upper probability of failure, limit state functions, credal sets, parameterized probability measures.

References

[1] Th. Fetz. Modelling uncertainties in limit state functions. *Int. J. Approx. Reasoning*, 53(1):1–23, 2012.