A Step Towards a Conflicting Part of a Belief Function on Three-element Frame of Discernment*

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Abstract

Frequently, belief functions [12] usually contain an internal conflict. Based on Hájek-Valdés algebraic analysis of belief functions [10], a unique decomposition of a belief function into its conflicting and non-conflicting part was introduced at ISIPTA'11 symposium for belief functions defined on two-element frame of discernment [3]. This contribution studies the conditions under which such a decomposition exists for belief functions (BFs) defined on three-element frame. For all necessary basic notions, illuminative figures, and references see [3].

When combining belief functions, a more complicated conflict often appears. Commonly used interpretation of the sum of conflicting masses $m_{\bigcirc\!\!\!\!\cap}(\emptyset)$ as a conflict between BFs is not correct. The problem of this interpretation was mentioned by Almond in 1995 [1], further by Liu [11], but the nature of the conflict has not been captured.

In [2, 7, 8], new ideas concerning interpretation, definition, and measurement of conflicts of BFs were introduced. An important difference between conflicts between BFs and internal conflicts of single BFs was pointed out; further, a conflict between BFs was distinguished from the difference/distance between BFs. When analyzing mathematical properties of the three approaches to conflicts of BFs from [2], there appears a possibility of expression of a BF Bel as Dempster's sum of non-conflicting BF Bel_0 with the same plausibility decisional support as the original BF Bel has and of indecisive BF Bel_S which does not prefer any of the elements of frame of discernment.

As only structures are described in the introduction to generalization of Hájek-Valdés analysis of BFs [5, 6], this study begins with an effort to make a generalization of Hájek-Valdés operation $-(a,b) = (b,a)$ and of the important homomorphism $f : (D_0, \oplus, -, 0, 0') \longrightarrow (S, \oplus, -, 0)$ given by $f(a,b) = (a,b) \oplus -(a,b)$, where $\oplus$ is Demspter's rule of combination.

Considering function '$-$' as transposition (permutation) of bbms of elements of the frame of discernment, we have $f(a,b) = (a,b) \oplus (b,a)$ as Dempster's sum of all permutations of bbms of $Bel = (a,b)$ on Ω_2. Analogously we can define

$$f(Bel) = \bigoplus_{\pi\in\Pi_3} \pi(Bel)$$

where Π_3 is the set of all permutations of bbms of elements of Ω_3: $\Pi_3 = \{\pi_{123}, \pi_{213}, \pi_{231}, \pi_{132}, \pi_{312}, \pi_{321}\}$, i.e., $f(a,b,c,d,e,f;g) = \bigoplus_{\pi\in\Pi_3} \pi(a,b,c,d,e,f;g) = (a,b,c,d,e,f;g) \oplus (b,a,c,d,f,e;g) \oplus (b,c,a,f,d,e;g) \oplus (a,c,b,e,d,f;g) \oplus (c,a,b,e,f,d;g) \oplus (c,b,a,f,e,d;g)$. It was proven that this is really homomorphism $f : D_3 \longrightarrow S$ of Dempster's semigroup $\mathbf{D}_3$ to its subsemigroup $S = (\{(a,a,a,b,b,b;1-3a-3b)\}, \oplus)$.

Having this, a series of open questions appears which are related to relation of this generalization of f to the partial generalization using $-Bel_0$ constructed via group G_3 of Bayesian BFs on Ω_3 from [3], see the updated schema of decomposition on Fig. 2. Further, the necessity of analysis of S_{Pl}, i.e., of subsemigroup of general indecisive belief functions, see Fig. 1, has appeared. Besides these new open questions, a partial positive result was reached: a unique decomposition for special classes of quasi Bayesian BFs.

Keywords. Belief function, Dempster-Shafer theory, Dempster's semigroup, conflict between belief functions, uncertainty, non-conflicting part of belief function, conflicting part of belief function.

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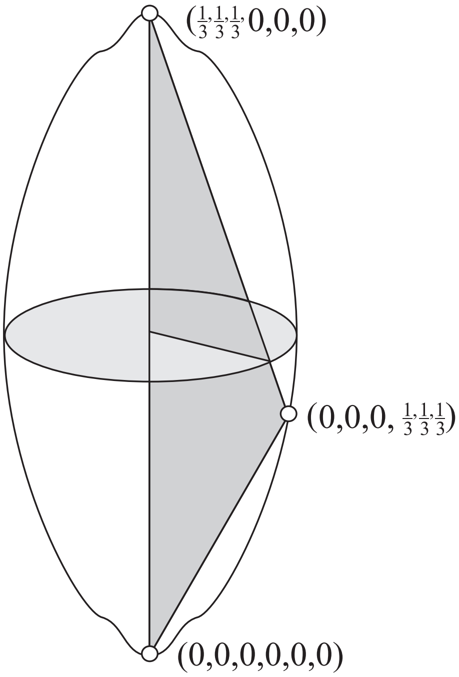

Figure 1: S_{Pl} — subsemigroup of general indecisive belief functions.

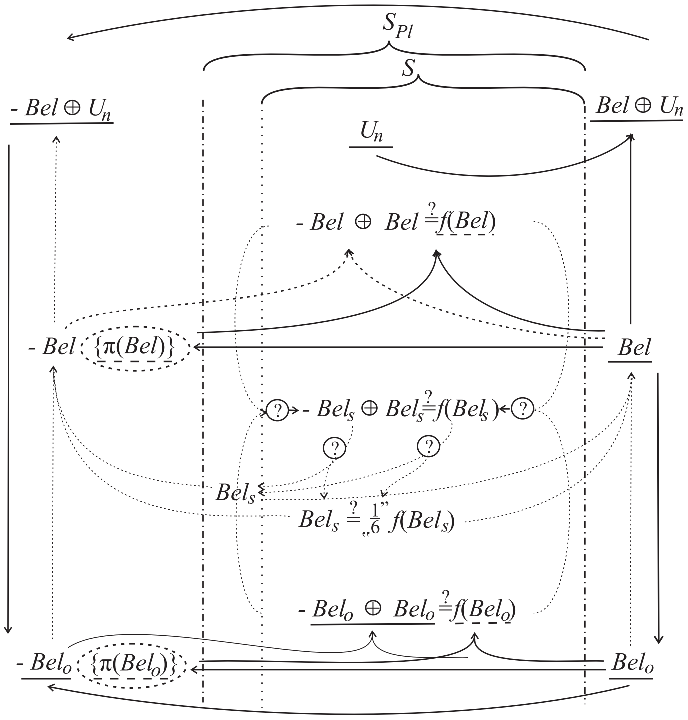

Figure 2: Updated detailed schema of a decomposition of BF Bel.

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